

and

$$H_{2n}^*(x) = \frac{H_{2n}(x)}{B_{2n}}, H_{2n-1}^*(x) = \frac{H_{2n-1}(x)}{B_{2n}}$$

where  $A_n = 2^n \Gamma[1 + (n/2)]$  and  $B_{2n} = (-1)^n [(2n)!/n!]$ . Here a very successful choice of the coefficients  $A_n$  and  $B_{2n}$  which permits a quick variation of integral error functions and Hermite's polynomials should be noted. The tables consist of two parts. The first part covers the functions of  $I_n \operatorname{erfc} x$  and the second those of  $H_n^*(x)$ . At the beginning of both parts of the tables the coefficients  $A_n$  and  $B_{2n}$  are given with nine-valued digits. All the functions are calculated with each step by  $x$  equal to 0.01. The function of  $I_0 \operatorname{erfc} x$  represents a separate table with six-valued digits for the values of  $x$  from 0 to 3.5. The functions of  $I_n \operatorname{erfc} x$  are given:

- (a) at  $0 < x < 1.0$  and  $1 < n < 30$   
with six-valued digits;
- (b) at  $1.01 < x < 1.5$  and  $1 < n < 25$   
with six-valued digits;
- (c) at  $1.51 < x < 2.0$  and  $1 < n < 15$   
with five-valued digits;
- (d) at  $2.01 < x < 2.5$  and  $1 < n < 10$   
with five-valued digits;
- (e) at  $2.51 < x < 3.0$  and  $1 < n < 5$   
with four-valued digits;
- (f) at  $3.01 < x < 3.5$  and  $1 < n < 3$   
with four-valued digits.

At  $0 < x < 10$  for all values of  $1 < n < 30$  Hermite's polynomials  $H_n^*(x)$  [ $H_0^*(x) = 1$ ] are given with six-valued digits.

Undoubtedly, the tables reviewed will be of great use to a broad section of scientific workers and engineers as well as for the students of senior courses studying the numerical solution of various problems on applied mathematical analysis.

A. V. LUIKOV

**Integral Transformations and Operational Calculus,**  
V. A. DITKIN and A. P. PRUDNIKOV, State Publishing House of Physical and Mathematical Literature, Moscow, 1961 (*Integral'nye preobrazovaniya i operatsionnoe ischislenie*, Gosudarstvennoe Izdatel'stvo Fiziko-Matematicheskoi Literatury), 524 pp.

ONE should welcome the appearance of this book *Integral Transformations and Operational Calculus* by V. A. DITKIN and A. P. PRUDNIKOV. For the first time there is published a book which systematically expounds various information in the theory of integral transformations and operational calculus.

The publication of the book was extremely necessary since the methods of integral transformations and operational calculus are highly effective methods of applied mathematical analysis which in many cases permit, with the help of simple rules, the solution of

complicated mathematical problems in various fields of modern natural science. These methods were successfully applied to mathematical physics; the theory of special functions; the calculation of integrals and summing functional series as well as to some problems of the number theory and so on. Operational methods play very important rôles in some modern branches of science and techniques such as: automation and telemechanics, the theory of follow-up systems, the heat transfer theory, atomic energetics etc. From this short summary it is clear that the publication of this book dedicated to integral transformations and operational calculus is a great event for a wide field of specialists, including both mathematicians and physicists, in the widest application of these terms.

The book contains the results of numerous investigations published in many periodicals and other works as well as a number of results obtained by the authors themselves. The book under review consists of two parts. The first one is devoted to the fundamentals of the theory and has five chapters. The first chapter deals with the elements of the theory of the Fourier transformations and some of their applications.

The second chapter which is dedicated to the Laplace transformation is the main and most extensive one. Here the Mellin transformation is also considered. Chapter 3 covers the Bessel integral transformation. A number of integral transformations based on the Bessel functions are referred to. In particular, the transformations of Hankel, Meyer and Lebedev-Kontorovich are investigated in this chapter. Chapter 4 gives a brief summary of the transformations of Meller-Fock, Hilbert and Laguerre. Chapter 5 deals with operational calculus. As is known, the desire to give a rigorous mathematical substantiation of the Heaviside formal rules led to the fact that operational calculus is expounded on the basis of the Laplace integral transformation; and the initial operator fundamental of operational calculus was replaced by the theory of functions of a complex variable with a wide application of contour integrals. However, the operator is one of the most common mathematical concepts, and the theory of linear operators is the most developed part of functional analysis and represents the basic apparatus of mathematical physics.

Such an extensive development of the operator theory also influenced the development of operational calculus. Absolute return to the viewpoint of the initial operator was made by Mikusinsky. He gave a rigorous operator substantiation of the Heaviside operational calculus and excluded the Laplace integral from the substantiation of operational calculus and, consequently, lifted restrictions connected with the behaviour of the considered functions at infinity. However, the elimination of the Laplace integral made it difficult to study the field structure of operators, since the Laplace integral is a natural way of presenting, by the functions of a complex variable, a field of operators. In the book reviewed this shortcoming was eliminated by introducing the Laplace generalized transformation with the help of which the Mikusinsky operator field reflects itself on a field in

an isomorphous way. Some sub-multiplicities formed by the functions of a complex variable are the elements of this field. Expounding the operational calculus, Mikusinsky had to introduce various designations for the function and its values at some point. In the present book the convolution, in contrast to Mikusinsky, is defined in such a way that there is no need to distinguish constants from their functions. Since in some cases, when applying the Laplace integral, the various transformations and calculations connected with the determination of operational formulas become considerably simplified, one may note the relation between a constructed calculus and the Laplace transformation. This same chapter introduces concepts of an operator function, the limit of sequence of operators, the limit of an operator function. Power functions, difference equations, operator differential equations and asymptotic series are considered. New operational calculus for the operator

$$B = \frac{d}{dt} + \frac{d}{dt}$$

caused by the Bessel equation, is given at the end of this chapter. For this operator basic operational relations are obtained and its application to solving some analytical problems are given. The relation between the new operational calculus and the Meyer transformation is established.

The second part of the book includes the tables of formulae of integral transformations which are widely applied to the most diverse branches of knowledge. A list of symbols of special functions and of some constants is given before the tables of formulae. Then tables of formulae of the following integral transformations: Fourier cosine and sine transformation, transformations of Laplace-Carson, Mellin, Hankel, Meyer, Kontorovich-Lebedev etc. are cited.

At the end of the book there is an extensive bibliography which covers almost all the problems concerning the theory of integral transformations and operational calculus as well as their numerous applications (270 titles).

Summarizing the review of the book, one should say that the authors were the first who in such a volume managed to systematize the information on integral transformations and operational calculus. One can say with certainty that this book will be a handbook for a broad section of specialists such as: mathematicians, physicists and engineers interested in problems of applied mathematics.

A. V. LUIKOV

**Evaporative Cooling of Circulating Water.** L. D. BERMAN, Pergamon Press, Oxford, 1961, 392 pp. 140s.

THE object of this book, translated from the 1956 edition of the Russian text first published in 1949, is to provide a systematic account of the theory, design and use of cooling methods for water, particularly in connexion with steam power plant. Most of the material is concerned with forced- and natural- convection cooling towers,

but attention is also paid to cooling in open reservoirs and in spray tanks.

The merit of such a volume can be judged in two ways: by consideration of the extent to which it aids the design of equipment; and by comparison with other published works on the same subject. These two methods will be used in turn in the present review, consideration being confined to cooling towers.

The task of designing a cooling tower is as follows: the mass flow rate of water and its inlet and outlet temperatures are specified, together with the temperature and humidity of the available cooling air; the tower shape and the type and quantity of packing have to be chosen, together perhaps with the fan capacity; and the capitalized cost of the plant has to be kept to a minimum. The design calculation has two phases: the analysis of the performance of a selected tower-plus-packing assembly to meet the required specification; and the systematic variation of the design parameters to minimize the cost. In both these phases the designer needs, on the one hand, appropriate mathematical methods, and, on the other, tabulations of data such as packing pressure drops, heights of a transfer unit, and costs of construction.

The book under review is definitely weak on the economic side. Although a few scattered cost data are given, no optimization methods are presented. The technical aspects of tower analysis are treated much more fully; for example, heat transfer coefficients and pressure drops are tabulated for a great many packing types, although unfortunately they are not tabulated side-by-side so that the reader has to work hard in order to assess the merit of a particular packing. This reviewer was disappointed not to find the "transfer unit" concept mentioned or used, although of course it appears implicitly in the equations which are solved. Incidentally, the differential equations for counterflow towers are studied in almost excessive detail; three slightly different methods of solution are presented, all of them making use in one way or another of the technique of linearization. Cross-flow towers are also treated, but without derivation of the solution presented.

The designer of a cooling tower has to pay great attention to the pressure drop and distribution of the air stream, a difficult task when the air enters in a horizontal direction at the base of the tower and must then turn through a right angle before flowing through the packing. The author of the book devotes several pages to the air-entry problem, writing down the equations governing the flow and presenting experimental data for various air-inlet arrangements; unfortunately the equations are not solved, nor could they be in the absence of values for the coefficients appearing in them; and the experiments are not reported in sufficient detail for the designer to be able to apply their results to any particular problem. This is altogether a tantalizing passage.

When the cooling tower is of the natural-draught type common in Europe, the task of designing it is particularly difficult, since the air-flow rate is dependent in a complex way on the tower dimensions, packing characteristics and inlet air and water conditions. Although Professor Berman discusses such towers at some length, he never